# SA to optimize the Multi-Period Water Distribution Network Design

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**Abstract.** The design optimization of the water distribution network is an important issue in modern cities. For this reason, we propose an optimization technique based on Simulated Annealing (SA) to solve the Water Distribution Network Design problem, considering multi-period restrictions with time varying demand patterns. The SA is improved with a local search procedure, yielding a hybrid SA, in order to obtain good quality networks designs. A broad experimentation using different test networks is carried out to test our proposal. Moreover, a comparison with an approach from the literature reveals the goodness of our proposal, the hybrid SA.

**Keywords:** Water Distribution Network Design, Optimization, Metaheuristic, Simulated Annealing

## 1 Introduction

In any modern city, a safe, adequate, and accessible supply of potable water is one of the basic necessities of any human being. In order to satisfy this vital requirement, enough volumes of water must be transported from a source of potable water, e.g., a treatment plant, to demand points (consumers) through a network of pipes. It is very important that the solution concerning the layout, design, and operation of the network of pipes result from good planning and management procedures. Consequently, this problem known as Water Distribution Network Design (WDND) requires to manage an important number of variables (pipes, pipe diameters, demand nodes, water pressure, reservoirs, etc.) and constraints (water velocity, pressure, etc.). This problem, even for simple networks, is very difficult to solve, in particular it is classified as NP-hard [1].

Early research works in the WDND optimization area were focused on the single-period, single-objective, gravity-fed design optimization problem. The first research works applied linear programming [2, 3], and non-linear programming [4, 5]. After that, the metaheuristics have been used to solve these problems, such as the trajectory-based ones: Simulated Annealing [6, 7] and Tabu Search [8]. Also population-based metaheuristics were applied, for example, Ant Colony Optimization [9], Ant Systems [10], Genetic Algorithms [11–13] Scatter Search [14], and Differential Evolution [15].

More recent works extends the single-period problem to a multi-period setting in which time varying demand patterns occur. In [16], the authors formulate the design problem as a multi-objective optimization problem and apply a multi-objective evolutionary algorithm. A Genetic Algorithm is used to solve six small instances considering velocity constraint on the water flowing through the distribution pipes in [12]. This constraint is also taken into account in [17], but the authors use mathematical programming on bigger, closer-to-reality instances. In [18], an Iterative Local Search is specifically-designed in order to consider that every demand node has 24 hrs water demand pattern and a new constraint, which imposes a limit on the maximal velocity of water through the pipes. Based on this last problem formulation, we propose a optimization technique in order to improve and optimize the distribution network design.

In this paper, we present a Simulated Annealing algorithm to optimize the optimal type of pipe connecting the supply, demand, and junction nodes in the distribution network. This SA incorporates a local search procedure in order to improve the layout of the network, arising the Hybrid Simulated Annealing (HSA) proposed. We test the performance of our proposal with a set of different networks with different sizes expressed by number of pipes and characteristics. The evaluation considers relevant aspects such as efficiency and internal behavior. Moreover, a comparison with state-of-the-art is carried out to outline the goodness of our proposal.

The rest of this article is organized as follows. In Section 2, we introduce the problem definition. Section 3 explains our algorithmic proposal, HSA, to solve the Water Distribution Network Design Optimization Problem. Section 4 describes the experimental analysis and the methodology used. Then, we analyze the results obtained by HSA and compare with the obtained by the ILS [18], in Section 5. Finally, we present our principal conclusions and future lines of research.

## 2 Multi-Period Water Distribution Network Design

This work focuses on obtaining the minimum cost in a water distribution network design. The problem can be characterized as: simple-objective, multi-period, and gravity-fed. Two restrictions are considered: the limit of water speed in each pipe and the demand pattern that varies in time.

The objective of the WDND problem is to minimize the total investment cost (TIC) of a water distribution network. The network can be modeled by a connected graph, which is described by a set of nodes $N = \{n_1, n_2, ...\}$, a set of pipes $P = \{p_1, p_2, ...\}$, a set of loops $L = \{l_1, l_2, ...\}$, and a set of commercially available pipe types $T = \{t_1, t_2, ...\}$ . The TIC is obtained by the formula shown in Equation 1.

$$\min TIC = \sum_{p \in P} \sum_{t \in T} L_p IC_t x_{p,t} \tag{1}$$

where $IC_t$ is the cost of a pipe $p$ of type $t$, $L_p$ is the length of the tube, and $x_{p,t}$ is the binary decision variable that determines whether the tube $p$ is of type $t$ or not. The objective function is limited by: physical laws of mass and energy conservation, minimum pressure demand in the nodes, and the maximum speed in the pipes, for each time $\tau \in \mathcal{T}$. These laws are explained in the following paragraphs.

**Mass conservation law:** It must be satisfied for each node $N$ in each period of time $\tau$. This law establishes that the volume of water flowing towards a node in a unit of time must be equal to the flow that leaves it (see Equation 2).

$$\sum_{n_1 \in N/n} Q_{(n_1,n),\tau} - \sum_{n_2 \in N/n} Q_{(n,n_2),\tau} = WD_{n,\tau} - WS_{n,\tau} \quad \forall n \in N \quad \forall \tau \in \mathcal{T} \tag{2}$$

where $Q_{(n_1,n),\tau}$ is the flow from node $n_1$ to node $n$ at time $\tau$, $WS_{n,\tau}$ is the external water supplied and $WD_{n,\tau}$ is the external water demanded.

**Energy conservation law:** It states that the sum of pressure drops in a closed circuit in an instant of time $\tau$ is zero. These drops can be approximated using the Hazen-Williams equations with the parameters used in EPANET 2.0 [19] (the hydraulic solver used in this paper), as indicated in Equation 3.

$$\sum_{p \in l} \left[ \frac{10.6668 y_{p,\tau} Q_{p,\tau}^{1.852} L_p}{\sum_{t \in T} (x_{p,t} C_t^{1.852} D_t^{4.871}} \right] = 0 \quad \forall l \in L \quad \forall \tau \in \mathcal{T} \tag{3}$$

In Equation 3, $y_{p,\tau}$ is the sign of $Q_{p,\tau}$ that indicates changes in the water flow direction relative to the defined flow directions, $Q_{p,\tau}$ is the amount of water flowing through pipe $p$ in time $\tau$, $L_p$ is the pipe length, $C_t$ is the Hazen-Williams roughness coefficient of pipe type $t$, and $D_t$ is the diameter of pipe type $t$.

**Minimum pressure head requirements:** for each node $n$ in each period of time $\tau$, it must be satisfied (see Equation 4).

$$H_{n,\tau}^{min} \leq H_{n,\tau} \quad \forall n \in N \quad \forall \tau \in \mathcal{T} \tag{4}$$

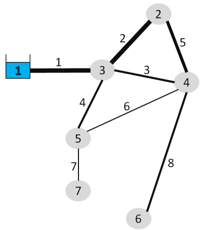
being $H^{min}$ the minimum node pressure and $H_{n,\tau}$ the node's current pressure.

**Maximum water velocity:** The water velocity $v_{p,\tau}$ can not exceed the maximum stipulated speed $v_{p,\tau}^{max}$. Equation 5 shows this relationship.

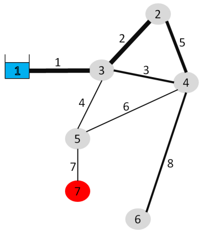
$$v_{p,\tau} \leq v_{p,\tau}^{max} \quad \forall p \in P \quad \forall \tau \in \mathcal{T} \tag{5}$$

## 3 Our Proposal for the Multi-Period WDND Problem

Simulated Annealing (SA) is a simple trajectory-based metaheuristic [20], which is based on a thermal process for obtaining low energy states of a solid in a heat bath. At the beginning (with a high temperature), SA accepts solutions with high cost values under a certain probability in order to explore the search space and to escape from local optima. During the annealing process this probability



(a)



(b)

| Pipe ID | Length (m) |
|---|---|
| 1 | 31 |
| 2 | 20 |
| 3 | 35 |
| 4 | 37 |
| 5 | 24 |
| 6 | 50 |
| 7 | 12 |
| 8 | 65 |

(c)

| Diam. (mm) | Roughness unitless | Cost |
|---|---|---|
| 60 | 130 | 22 |
| 80 | 130 | 35 |
| 100 | 130 | 38 |
| 150 | 130 | 50 |
| 200 | 130 | 61 |

(d)

**Fig. 1.** Different solutions or network designs. (a) Solution 1; (b) Solution 2; (c) Pipe lengths; (d) Available pipe types with their corresponding costs.

**Table 1.** Different solutions or network designs in vector representation.

| Solution | Pipe ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Feasibility |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Length (m)** | **31** | **20** | **35** | **37** | **24** | **50** | **12** | **65** | **TIC** |
| 1 | diam. (mm) | 150 | 150 | 80 | 80 | 100 | 60 | 60 | 80 | feasible |
| | cost | 1550 | 1000 | 1225 | 1295 | 912 | 1100 | 264 | 2275 | 9621 |
| 2 | diam. (mm) | 150 | 150 | 80 | 60 | 100 | 60 | 60 | 80 | infeasible |
| | cost | 1550 | 1000 | 1225 | 814 | 912 | 1100 | 264 | 2275 | 9140 |

decreases according to temperature cooling; intensifying the search and reducing the exploration in order to exploit a restricted area of a search space.

Simulated annealing evolves by a sequence of transitions between states and these transitions are generated by transition probabilities. Consequently, SA can be mathematically modeled by Markov chains, where a sequence of chains is generated by a transition probability, which is calculated involving the current temperature.

Our proposal consists in adapting and hybridizing the SA algorithm to solve the Multi-Period WDND optimization problem. In this way, the Hybrid Simulated Annealing (HSA) algorithm arises. A solution to this problem is a network, as shown in figures 1 (a) and (b). A network or a solution is represented by a vector, where each element is the diameter selected for that pipe, as can be seen in Table 1. In this table the vectors that represent the candidate solutions in figures 1(a) and (b) are shown. The total investment cost for each solution is calculated by the Equation 1, using the input data from tables (c) and (d) of Figure 1. The first solution is hydraulically feasible (satisfying all constraints mentioned in Section 2) and the second one is infeasible (violating the minimum pressure constraint in node 7).

**Algorithm 1** HSA Algorithm to solve the WDND optimization Problem

1: $t = 0$;
2: initialize $T$ and $S_0$; {temperature and initial solution}
3: evaluate $S_0$ in $TIC_0$;
4: **repeat**
5:   **repeat**
6:     $t = t + 1$;
7:     generate $S_1$ from $S_0$ applying the MP-GRASP Local Search;
8:     evaluate $S_1$ in $TIC_1$;
9:     **if** $TIC_1 < TIC_0$ **then**
10:       $S_0 = S_1$; $TIC_0 = TIC_1$
11:     **end if**
12:     generate $S_2$ from $S_0$ applying the perturbation operator;
13:     evaluate $S_2$ in $TIC_2$;
14:     **if** $(TIC_2 < TIC_0)$ **or** $(exp((TIC_2 - TIC_0)/T) > random(0, 1))$ **then**
15:       $S_0 = S_2$; $TIC_0 = TIC_2$
16:     **end if**
17:   **until** ($t$ mod Markov Chain Length) == 0
18:   update $T$;
19: **until** *stop criterion is met*
20: **return** $S_0$;

In Algorithm 1, we show a pseudo-code of the HSA algorithm to solve the WDND optimization problem. HSA uses the EPANET 2.0 toolkit [19] to solve the hydraulic equations, since this hydraulic solver is applied in most existing works. HSA generates a feasible initial solution $S_0$ applying both `HighCost` and `Lowcost` mechanisms proposed in [18] (line 2). After the evaluation of the initial solution (line 3), an iterative process starts (lines 4 to 19). As a first step in the iteration, the hybridization is carried out in order to intensify the search into the current region of the solution space. In this way a feasible solution, $S_1$, is obtained by applying the MP-GRASP local search [18] to $S_0$ (line 7), and then a greedy selection mechanism is performed (lines 9-11). As a consequence, $S_0$ can be replaced by $S_1$ if this is better than $S_0$. In the next step a perturbation operator is used to obtain a feasible neighbor, $S_2$, from $S_0$ (line 12), in order to explore another areas of the search space. This perturbation randomly changes some pipe diameters. If $S_2$ is worse than $S_0$, $S_2$ can be accepted under the Boltzmann probability (line 14, second condition). In this way, at high temperatures ($T$) the exploration of the search space is strengthened. In contrast, at low temperatures the algorithm only exploits a promising region of the solution space, intensifying the search. In order to update $T$, the proportional cooling process [20] is used (line 18) and it is applied after a certain number of iterations ($t$) given by the Markov Chain Length ($MCL$) (line 17). Finally, SA ends the search when the total evaluation number ($\sharp Eval.$) is reached.

## 4 Experimentation and Results

In this section, we introduce the experimental design used in this approach, the execution environment, and the result analysis. In order to evaluate HSA, the HydroGen instances of WDND optimization problem [21] are solved. These instances arise from 15 different distribution networks (HG-MP-$i$), which information is shown in Table 2. A set of 16 different pipe types is used and their

**Table 2.** Information on the HydroGen networks.

| Network | Meshedness Coefficient | Pipes | Demand Nodes | Water Reservoirs | Network | Meshedness Coefficient | Pipes | Demand Nodes | Water Reservoirs |
|---|---|---|---|---|---|---|---|---|---|
| HG-MP-1 | 0.2 | 100 | 73 | 1 | HG-MP-9 | 0.1 | 295 | 247 | 2 |
| HG-MP-2 | 0.15 | 100 | 78 | 1 | HG-MP-10 | 0.2 | 397 | 285 | 2 |
| HG-MP-3 | 0.1 | 99 | 83 | 1 | HG-MP-11 | 0.15 | 399 | 308 | 2 |
| HG-MP-4 | 0.2 | 198 | 143 | 1 | HG-MP-12 | 0.1 | 395 | 330 | 3 |
| HG-MP-5 | 0.15 | 200 | 155 | 1 | HG-MP-13 | 0.2 | 498 | 357 | 2 |
| HG-MP-6 | 0.1 | 198 | 166 | 1 | HG-MP-14 | 0.15 | 499 | 385 | 3 |
| HG-MP-7 | 0.2 | 299 | 215 | 2 | HG-MP-15 | 0.1 | 495 | 413 | 3 |
| HG-MP-8 | 0.15 | 300 | 232 | 2 | | | | | |

**Table 3.** Available pipe types and their corresponding costs.

| Number | Diam. (mm) | Roughness | Cost | Number | Diameter | Roughness | Cost |
|---|---|---|---|---|---|---|---|
| 1 | 20 | 130 | 15 | 9 | 200 | 130 | 116 |
| 2 | 30 | 130 | 20 | 10 | 250 | 130 | 150 |
| 3 | 40 | 130 | 25 | 11 | 300 | 130 | 201 |
| 4 | 50 | 130 | 30 | 12 | 350 | 130 | 246 |
| 5 | 60 | 130 | 35 | 13 | 400 | 130 | 290 |
| 6 | 80 | 130 | 38 | 14 | 500 | 130 | 351 |
| 7 | 100 | 130 | 50 | 15 | 600 | 130 | 528 |
| 8 | 150 | 130 | 61 | 16 | 1,000 | 130 | 628 |

**Table 4.** Average of the best TIC values obtained by ILS and HSA for all instances grouped by the corresponding network. The best values are boldfaced.

| Network | ILS | HSA | Network | ILS | HSA |
|---|---|---|---|---|---|
| HG-MP-1 | 339200 | **335883** | HG-MP-9 | **831200** | 834821 |
| HG-MP-2 | 303800 | **298842** | HG-MP-10 | 790600 | **788422** |
| HG-MP-3 | 389200 | **387089** | HG-MP-11 | **907000** | 909038 |
| HG-MP-4 | 694200 | **690033** | HG-MP-12 | 1022800 | 1046426 |
| HG-MP-5 | 728200 | **722218** | HG-MP-13 | 1190400 | **1179568** |
| HG-MP-6 | 750400 | **741638** | HG-MP-14 | **1070200** | 1085195 |
| HG-MP-7 | 818000 | **817262** | HG-MP-15 | **1122600** | 1161200 |
| HG-MP-8 | **851600** | 855479 | | | |

characteristics and costs can be found in Table 3. The demand nodes are divided into five categories (domestic, industrial, energy, public services, and commercial demand nodes), each one with a corresponding base load and demand pattern[3]. In this way, five different instances are considered for each HG-MP-$i$ network, resulting 75 instances in total.

The computational environment used in this work to carry out the experimentation consists of computers with INTEL I7 3770K quad-core processors 3.5GHz, 8 GB RAM, and the Slackware Linux with 3.2.29 kernel version. Because of the stochastic nature of the algorithms, we performed 30 independent runs of each instance to gather meaningful experimental data and apply statistical confidence metrics to validate our results and conclusions. As a result, a total of 2,250 executions were carried out.

The methodology followed to analyze the results is described in the following. First, we study the HSA behavior, comparing the best cost values found by

[3] The base loads can be found in the EPANET input files of the instances

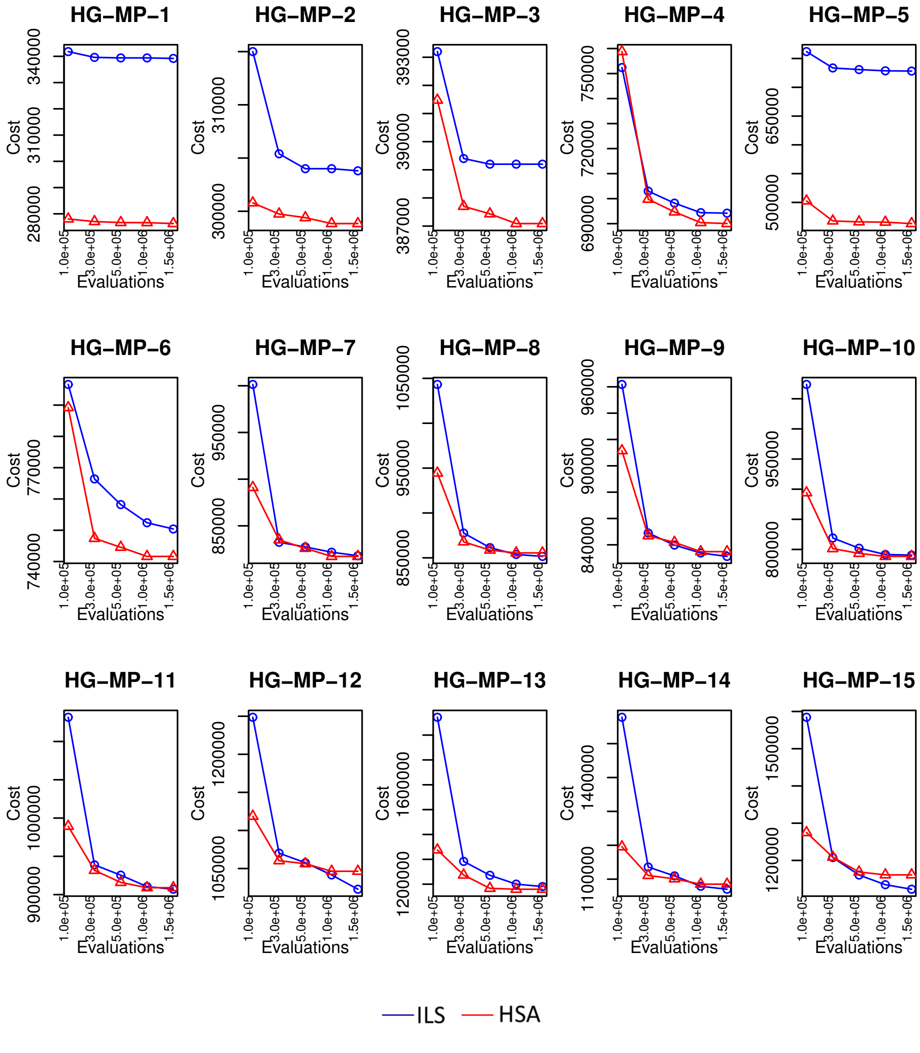



**Fig. 2.** Evolution of the TIC values during the search for all instances.

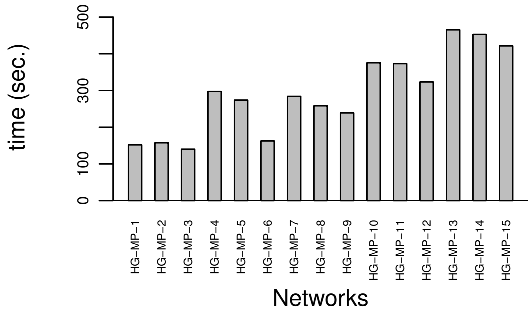



**Fig. 3.** Average of the total HSA's execution times for all instances grouped by the corresponding network.

our proposal and ILS [18] for the 75 instances grouped by their corresponding distribution network, as presented in Table 4. Secondly, we analyze the HSA convergence, in comparison with ILS, taking into account the cost values found at the 1e+05, 3e+05, 5e+05, 10e+05, and 15e+05 EPANET calls (evaluations), as shown in Figure 2. Besides, the HSA's execution times (in seconds) to carry out the maximum number of evaluations for each test case, grouped by network, are shown in Figure 3. Note that, we only present the HSA's total execution time for all test case, because no data about this metric are reported in [18].

Analyzing the Table 4, we detect that HSA found solutions with less TIC than the ones of ILS in nine networks. As a consequence, in 60% of the problem instances better cost values are found when they are solved by HSA. The HSA advantage arises out of the Boltzmann probability application to accept high TIC values, which allows diversify the search in order to escape from local optima.

From the convergence point of view, we observe that HSA found solutions with TIC near to the best ones in 80% of the instances, with only 1e+05 evaluations. Instead, ILS needed at least 3e+05 evaluations for that, besides this is achieved in only 66% of the test cases.

Evaluating both the Figure 3 and the Table 2 together, we notice that the HSA's execution time is affected by the number of pipes and demand nodes. In this way, five groups of three networks can be formed exhibiting similar execution times. These instances have consecutive numbers, e.g. the set of the HG-MP-1, 2, and 3 networks have similar number of pipes and demand nodes, and so on. Being the set formed by the HG-MP-13, 14, and 15 networks the most expensive cases to solve. Furthermore, analyzing what happened into each set of networks, the network with more demand nodes consumes less execution time than the other two, since more feasible solutions exists and HSA need less time to find one of them.

Summarizing, HSA outperformed ILS in the result quality of 60% of the problem instances. Moreover, a quick convergence to good solutions is also evidenced by our proposal in most of the problem instances. Furthermore, the HSA's runtime is affected by the growing and combination of the number of pipes and demand nodes.

## 5 Conclusions

In this paper, we have proposed an optimization technique based on the Simulated Annealing algorithm that can find successful water distribution network designs, considering multi-period settings with time varying demand patterns. This new technique, called HSA, solves the hydraulic equations by using the EPANET 2.0 toolkit. HSA combines an WDND-adapted SA with the MP-GRASP local search [18]. For this study, we have tested 75 instances that come from 15 different HydroGen networks.

The HSA's results were compared with the obtained by the ILS proposed in [18] to solve this problem. As a consequence, we observed that HSA outperformed the results obtained by ILS in more than half (60%) of instances, since HSA achieved a better exploration than ILS by using the Boltzmann probability to accept new solutions. This advantage combined with the local search allowed HSA to converge quickly on the best solutions.

For future works, we will be tackling the multi-period WDND optimization problem improving the HSA by introducing a specific heuristic into the perturbation operator. We are also interested in testing larger dimension instances, as close as possible to real scenarios.

## Acknowledgments

The authors acknowledge the support of Universidad Nacional de La Pampa and the Incentive Program from MINCyT. The second author is also funded by CONICET.

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